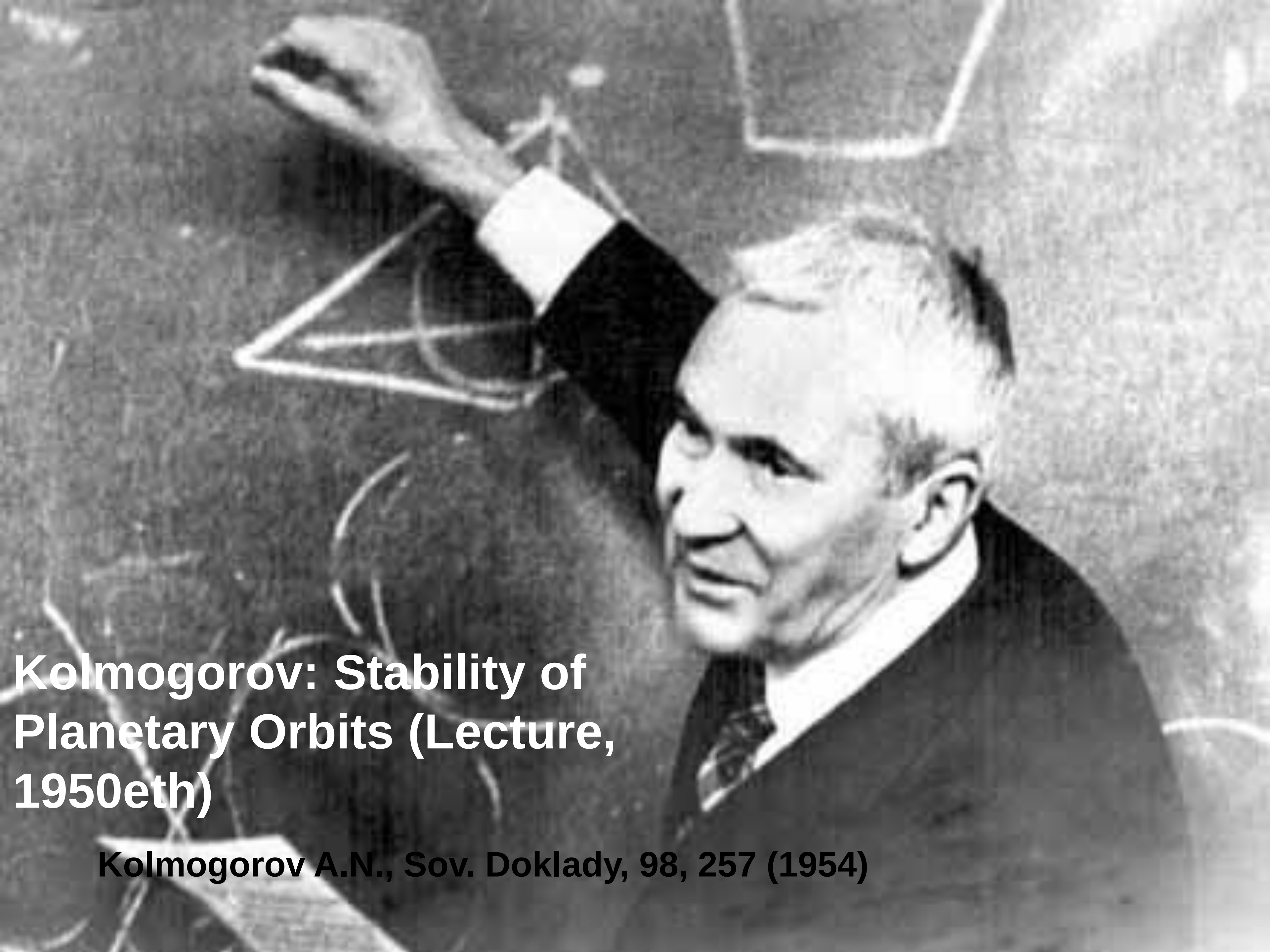


Kolmogorov: Stability of Planetary Orbits (Lecture, 1950eth)

Kolmogorov A.N., Sov. Doklady, 98, 257 (1954)

KAM-theorem, QLT of plasma, Chaos and beyond

- Everything started with “Stability of Solar System”
- KAM theory
- Did it make Planetary motion more stable?
- Quasilinear Theory is opposite limit re:KAM (Landau Resonances vs. Planetary Resonances)
- Hamiltonian Chaos

$$H = H_0(I) + \varepsilon V(I, \vartheta)$$

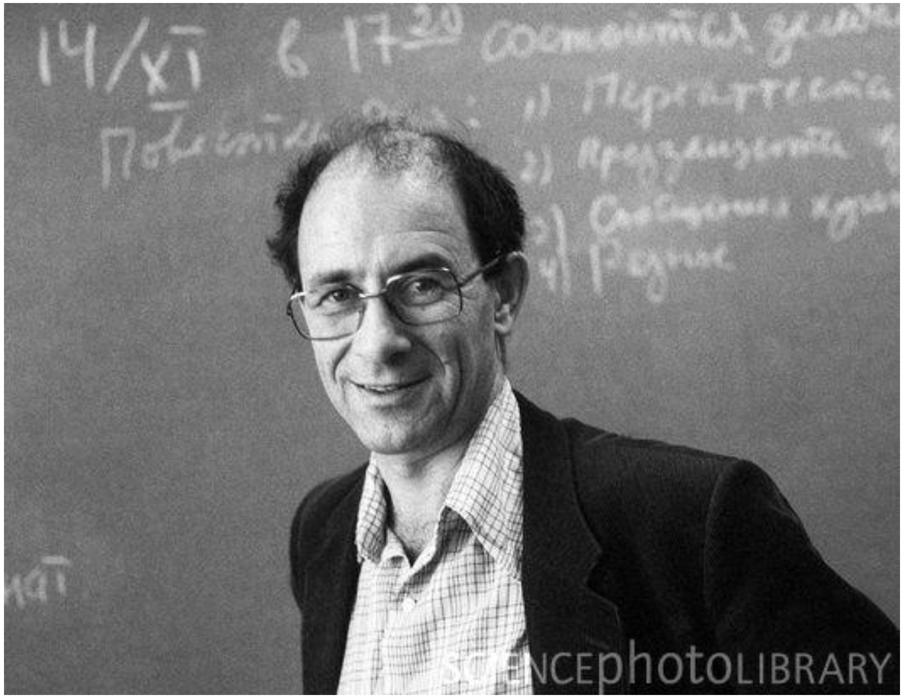
$$\omega(I) = \frac{\partial H_0}{\partial I}$$

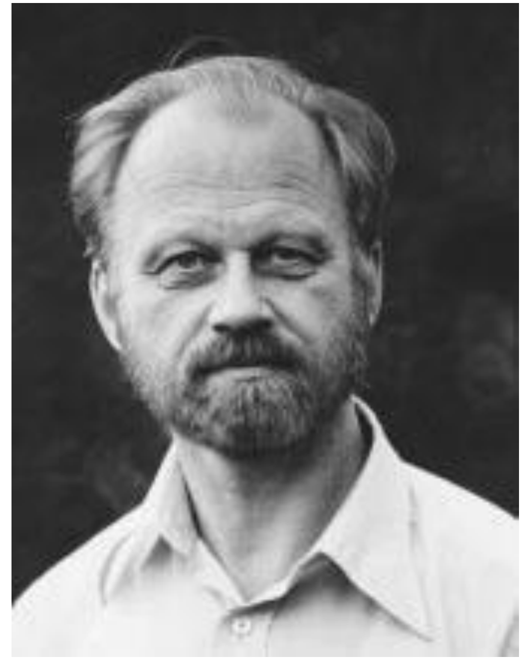
$$\Delta I \approx \varepsilon^{\frac{1}{2}}$$

Strength of Planet/Planet interaction

Width of resonant region

K + A and M added





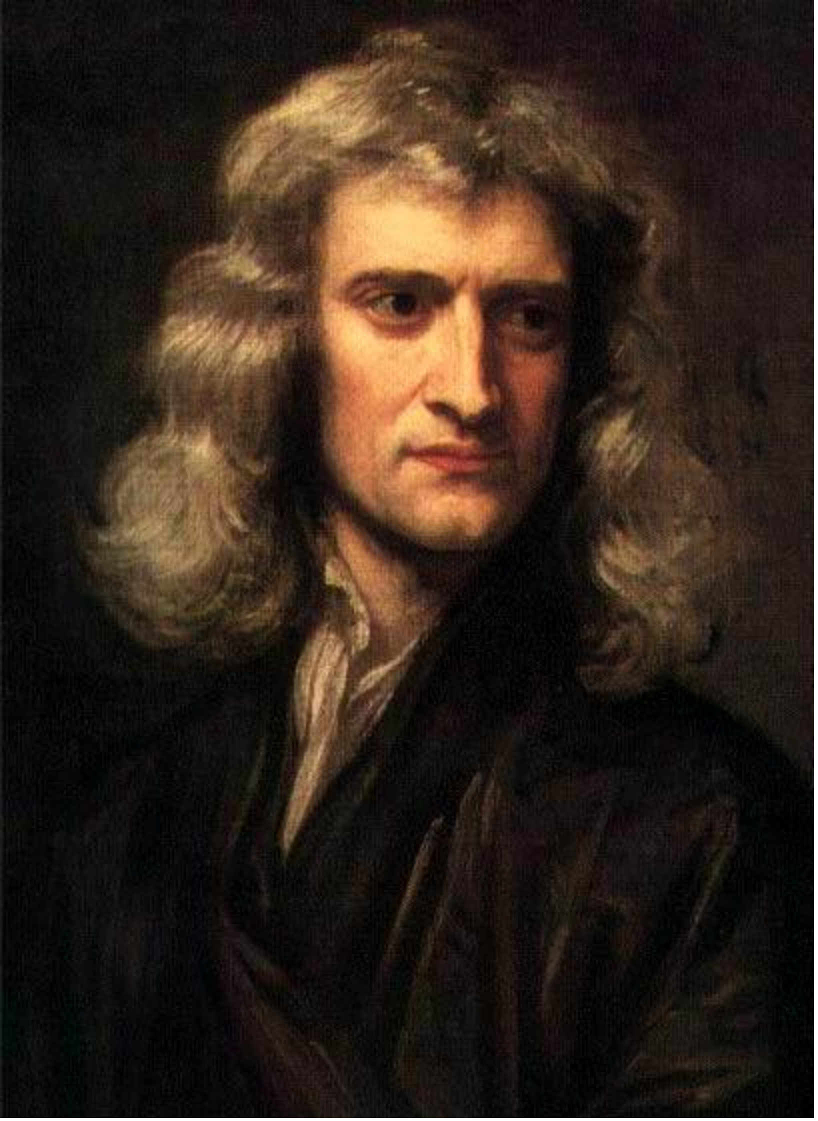
Arnold V.I., Izvestia of Sov. Acad.,

25, 25 (1961)

Moser J., Nachr. Acad. Wiss. Gottingen, Math Phys., K1, 11a,1 (1962)

Going beyond 2-body problem (adding planet/planet interaction)

- Newton
- Laplace
- Poincare
- Perturbation technique and extraction of secular effects
- Planetary resonances of higher orders
- KAM theorem



[***Philosophiæ Naturalis Principia Mathematica***]

1687

Newton/s conjecture - Solar System is UNSTABLE; needs DIVINE INTERVENTIONS (how frequently?)

Pierre-Simon Laplace
1749-1827

Méchanique céleste

Exposition du système du monde

``Je n'avais pas besoin de cette hypothèse-là”

Where do we stay today ?

- KAM –theorem is not exactly applicable to Solar System

- Search of secular effects with direct computer simulation

- Effects of multi-dimensionality (N>2, Arnold)

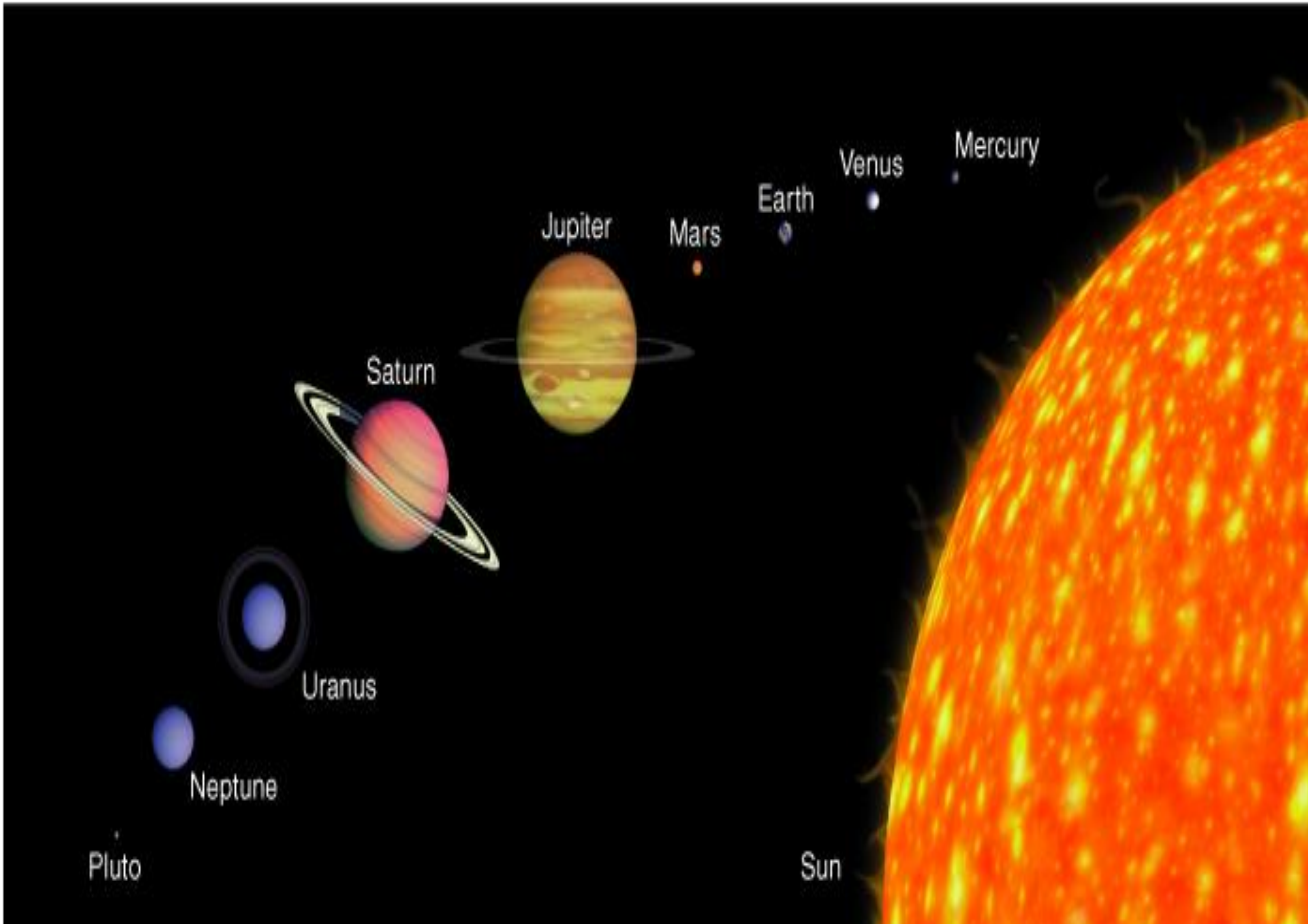
Modern view resulting from such computer simulation

- Endangered species (planets) identified: Pluto, Mercury; time scale for cataclysmic outcome 100 – 800 Mln years

- Should we be afraid ?

- IAU and Pluto

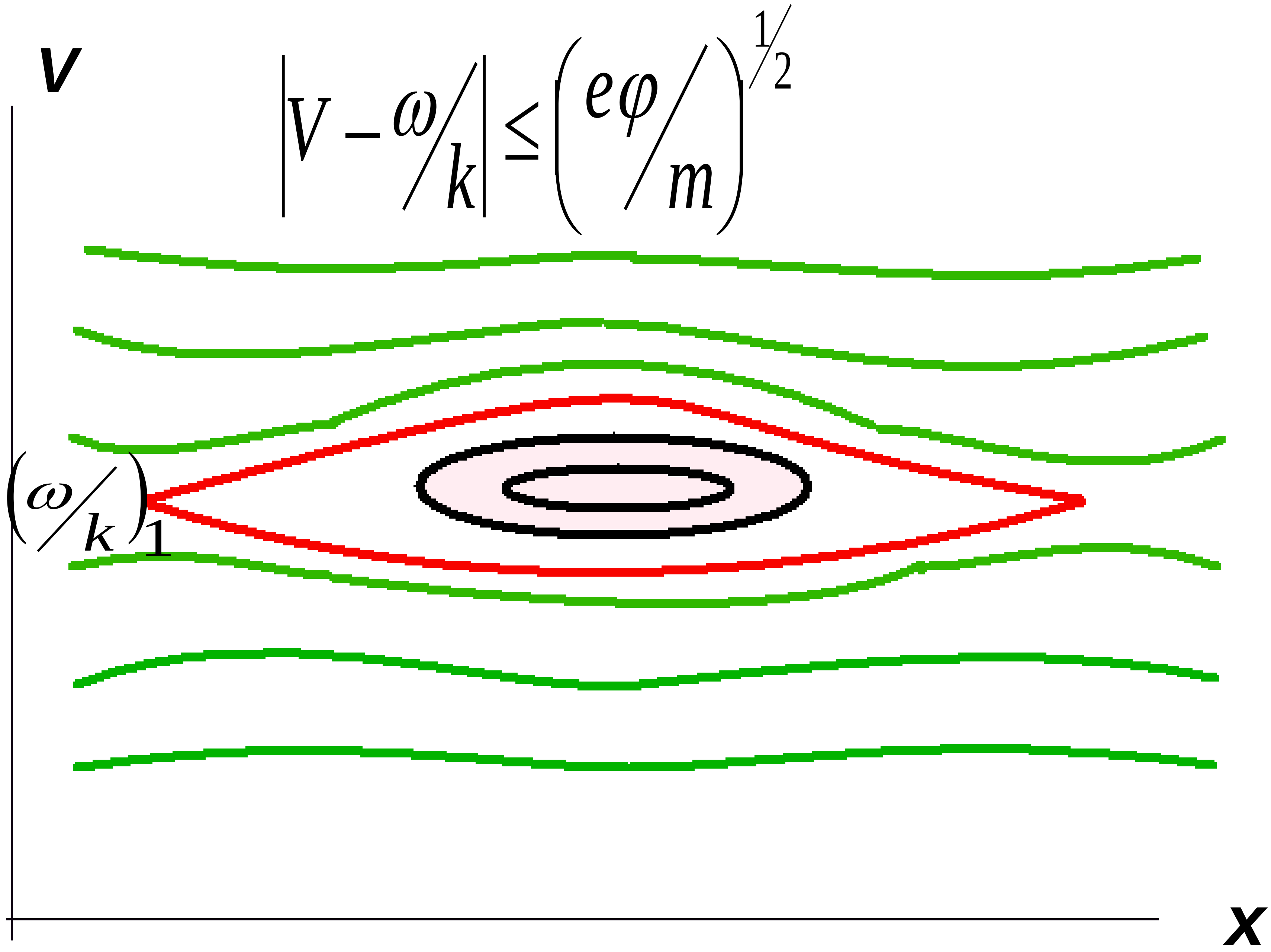
- Solar System might have had one more planet(?)

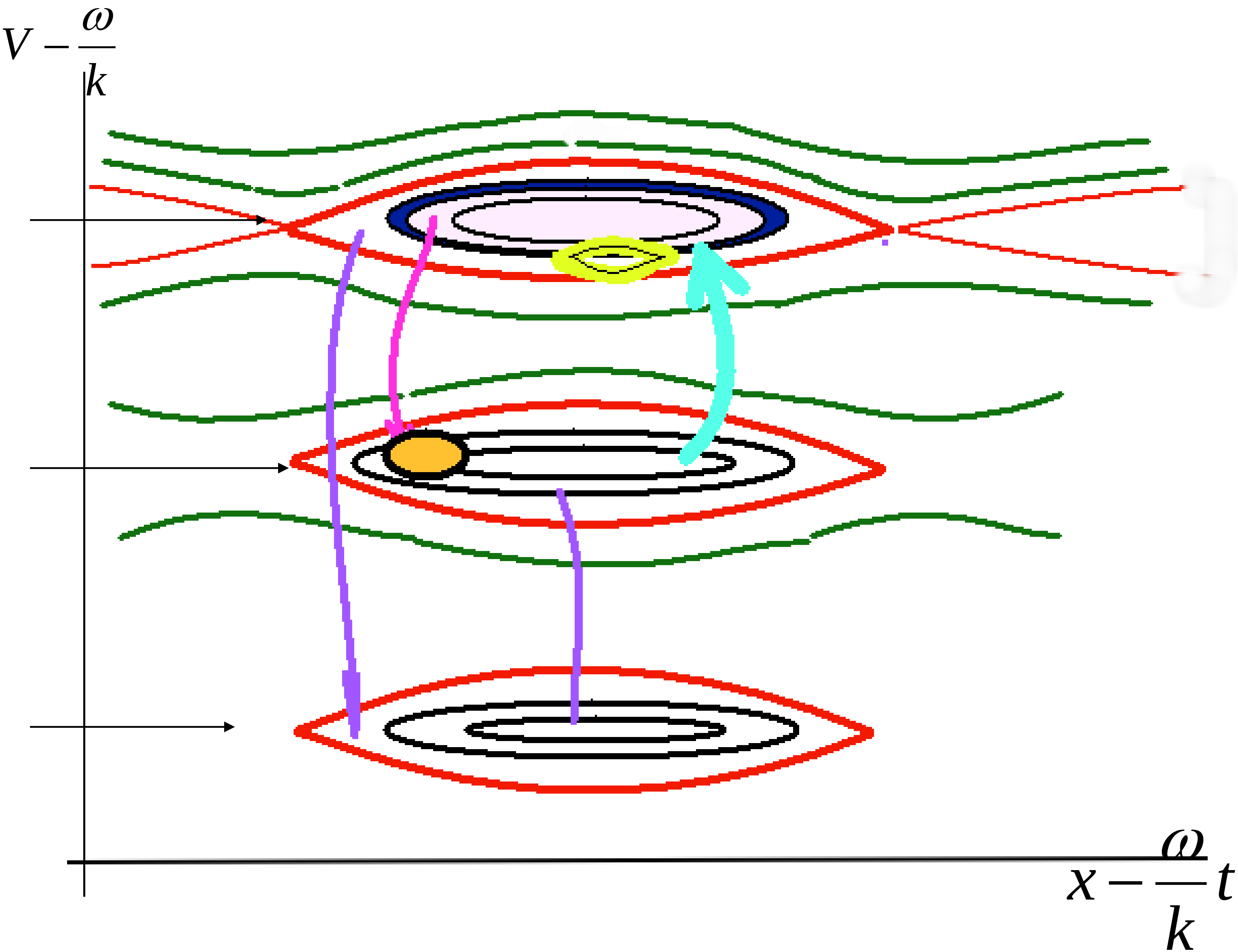

Mercury
Venus
Earth
Mars
Jupiter
Saturn
Uranus
Neptune
Pluto
Sun


IKI –Institute of Space Research, Moscow;
Image of the Nucleus of Halley's Comet by Vega S/C camera
orbits of comets might be unstable in much shorter time scale
(C) IKF
Smoothed

Back to KAM theorem and take Landau resonances instead planetary ones:

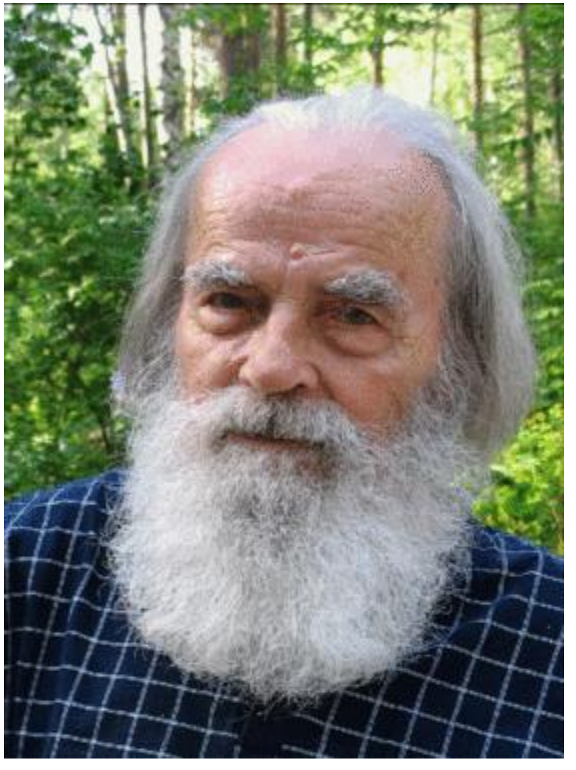
$$m\frac{dV}{dt} = e\sum E_i \exp i(\omega_i - kv)t$$


V
$|V - \omega/k| \leq \left(e\varphi/m\right)^{1/2}$
$(\omega/k)_1$
X



$V-\frac{\omega}{k}$
$x-\frac{\omega}{k}t$

Boris Chirikov and “Standard MAP”



Chirikov B.V., Phys. Rep., 52,263 (1979)

Atomic Energy (in Russian), 6, 630 (1959)

$$\left(\frac{e\varphi}{m} \right)^{1/2} \text{ much less than } \left(\frac{\omega}{k} \right)_{n+1} - \left(\frac{\omega}{k} \right)_n$$

This limit corresponds to KAM (Kolmogorov-Arnold-Mozer) case.

KAM-Theorem :

As applied to our case of Charged Particle – Wave Packet Interaction –

"Particle preserves its orbit "

$$\left(\frac{e\varphi}{m}\right)^{1/2} \quad \text{greater than} \quad \left(\frac{\omega}{k}\right)_{n+1} - \left(\frac{\omega}{k}\right)_{n}$$

That means - overlapping of neighboring resonances

Repercussions:

-"collectivization" of particles between neighboring waves;

-particles moving from one resonance to another – "random walk"? And if yes

- what is Diffusion Coefficient ?(in velocity space)

$$m\frac{dV}{dt} = e\sum E_i \exp i(\omega_i - kv)t$$

$$V = {}^{e}\!/_{m}\sum E_i \exp i(\omega - kv)t \Big/ i(\omega - kv)$$

***V*×*dV/dt* =**

$$e^2 \Big/ m^2 \sum\sum EE^* \exp i(\omega_i - \omega_j - k_i v + k_j v)t \Big/ i(\omega - kv)$$

$$V^2 \propto Dt$$

$$D = \pi e^2 / m^2 \sum |E|^2 \delta(kv - \omega)$$

$$\sum_k = \frac{1}{2\pi} \int dk$$

Quasilinear Theory is an example of

Anti - KAM limiting case (1961, Salzburg conference)

Repercussions: Quasilinear Theory, Plateau Formation,

Beam + Plasma Instability Saturation etc.

Extention of Quasilinear approach to Instability: Velocity Anisotropy ("Cyclotron Instability" of Alfven waves)

$$\omega + kv_z = \omega_B \quad \text{(Cyclotron resonance)}$$

$$\gamma \propto \int dv \left[\left(1 - kv_\perp/\omega\right) \frac{\partial f}{\partial v_\perp} + kv_\perp/\omega \frac{\partial f}{\partial v_z} \right]$$

(Sagdeev&Shafranov,1960)

$$\hat{D}_{QL}\, f =$$

$$(e/M)^2 \sum |E|^2 \delta(\omega - \omega') \left[(1 - k v_z/\omega)\, 1/v_\perp \, \frac{\partial}{\partial v_\perp} v_\perp + k v_z/\omega \, \frac{\partial}{\partial v_z} \right]$$

$$\mathrm{X} \left[(1 - k v_z/\omega) \frac{\partial f}{\partial v_\perp} + k v_\perp/\omega \frac{\partial f}{\partial v_z} \right]$$

$$\omega' = \omega_B + k v_z$$

(Same paper at Salzburg, 1961)

$$\hat{D}_{QL}\, f =$$

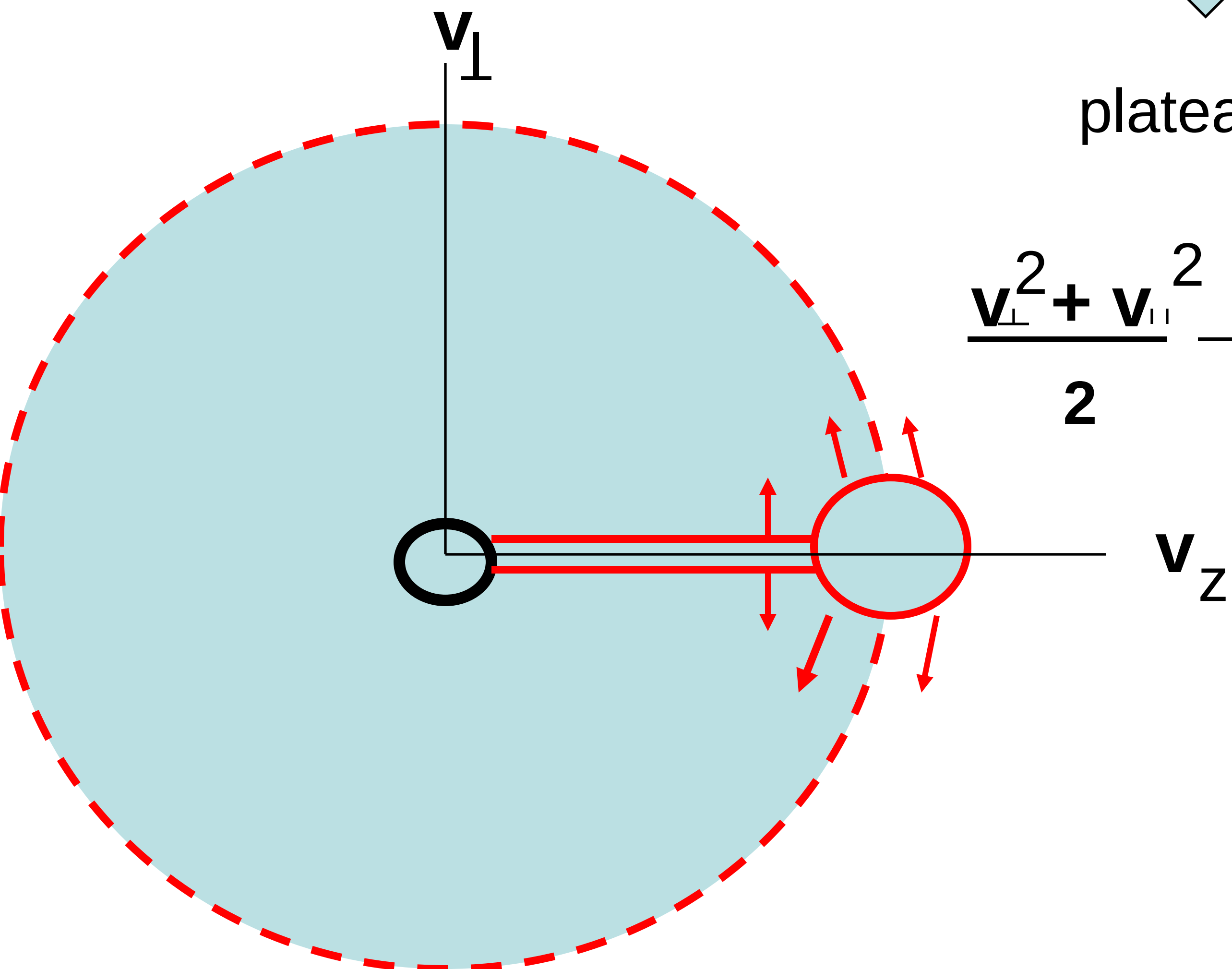
$$(e/M)^2 \sum |E|^2 \delta(\omega - \omega^l)[(\not{1} - kv_z/\omega)1/v_\perp \frac{\partial}{\partial v_\perp} v_\perp + kv_\perp/\omega \frac{\partial}{\partial v_z}]$$

$$\times [(\not{1} - kv_z/\omega)\frac{\partial \mathbf{f}}{\partial v_\perp} + kv_\perp/\omega \frac{\partial \mathbf{f}}{\partial v_z}]$$

$$\frac{1}{\omega_B} \frac{\partial}{\partial \vartheta} \left(\sum |\mathbf{B}|^2 \delta(\omega - \omega^l) \frac{\partial}{\partial \vartheta} \mathbf{f} \right)$$

(Kennel , Petchek, 1966)

Feedback on particles: Quasilinear Theory of Particles/Cyclotron Waves Interaction



plateau

$$\frac{v_\perp^2 + v_\parallel^2}{2} - \frac{\omega \, v_\parallel}{k} = \text{const}$$

If $\beta \gg 1$

Simplified approach

- Spatial Diffusion approximation is valid:

-QL estimate of $\nu_{eff} \approx \omega_B \frac{(\delta B)^2}{\mathbf{B}^2}$

$$\mathbf{L}_{eff} \approx \frac{\mathbf{C}}{\nu_{eff}} \ ;$$

l

Magnetic field lines diffusion

Rosenbluth M.N., Sagdeev R.Z.,

Taylor J.B., **Zaslavsky G.M**.,

Nucl. Fusion, 6, 297 (1966)

$$dx/B_x = dy/B_y = dz/B_z$$

Z plays role of time;

dB – role of wave amplitude

Web Map (Zaslavsky Map)

```
U'=V*Cos(Q)-(U+K*Sin(V+2*PI*F*N))*Sin(Q)
V'=V*Sin(Q)+(U+K*Sin(V+2*PI*F*N))*Cos(Q)
Q=2*PI/A
```

V'=V*Cos(Q)-(U+K*Sin(V+2*PI*F*N))*Sin(Q)

U'=V*Sin(Q)+(U+K*Sin(V+2*PI*F*N))*Cos(Q)

Q=2*PI/A

- PARAMETERS OF MAP -	
K	.130000E+00
A	.411000E+03
F	.250000E+00

THE CHANGING OF ALL VALUES
IS FOLLOWED BY START MAP FROM THE BEGINNING

----- PLOTTING PARAMETERS -----

STEP FOR PLOTTING= 1

START VARIABLES BY VALUE

START VARIABLES BY CURSOR

COLOR 15

COLOR INCREMENT= 0

START 'SHIFT X' = .000000E+00

START 'SHIFT Y' = .000000E+00

START 'SIZE X' =2*PI* .120000E+02

START 'SIZE Y' =2*PI* .899062E+01

PUT ZOOM

CURRENT 'SHIFT X' = .000000E+00

CURRENT 'SHIFT Y' = .000000E+00

CURRENT 'SIZE X' =2*PI* .120000E+02

CURRENT 'SIZE Y' =2*PI* .899062E+01

SCREEN PROPORTIONAL= YES

TORUS

LENGTH OF LINE= 0.0000000000000

--- MAP TYPES ---

WEB WAVE MAP

STANDARD

RELAT. BASIN

INHOMOGENEOUS

WEB 2-WAVE

DIFFUS 2-WAVE

Z-K

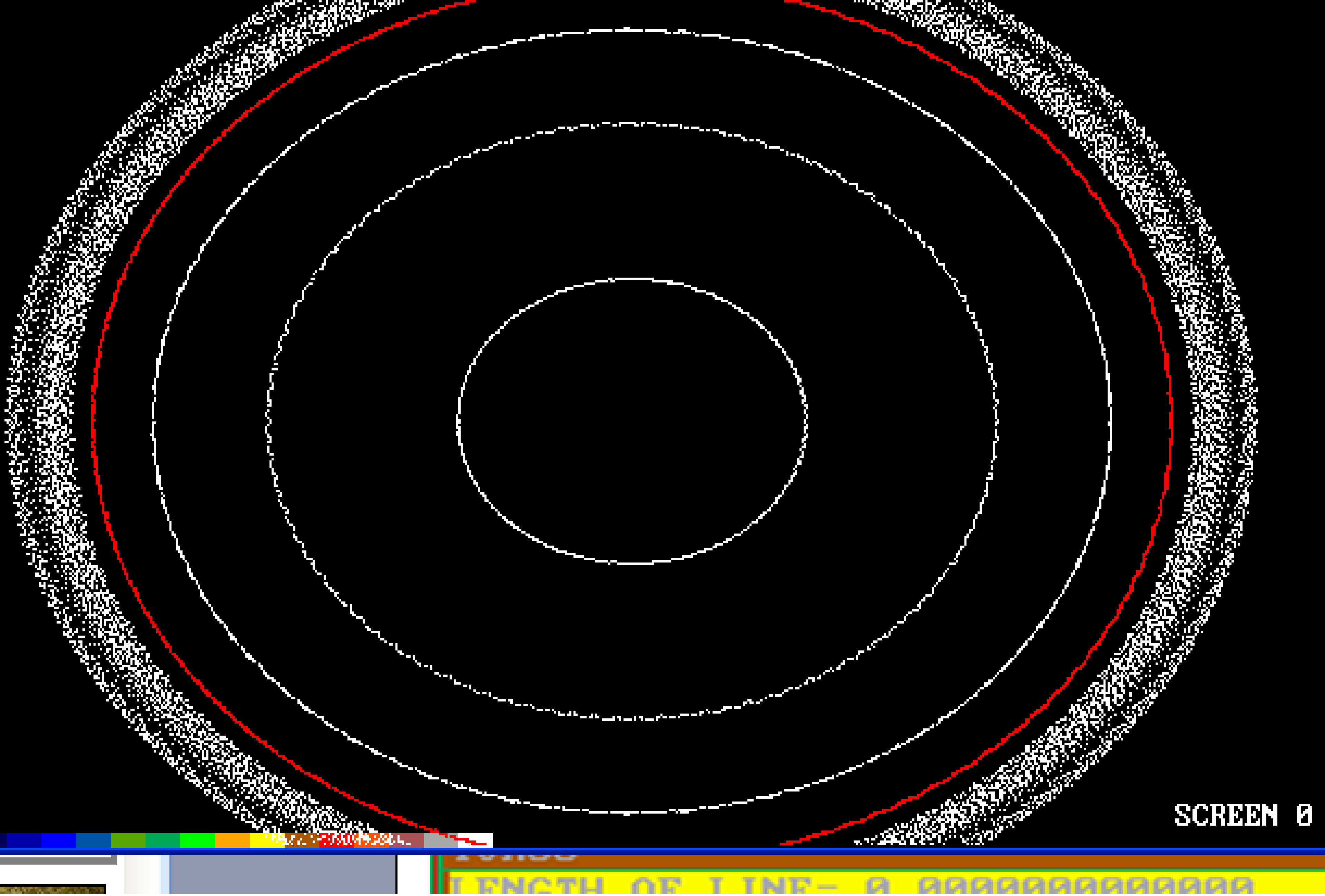
SUPERBALL

BAR

ANTI_STANDARD

STANDARD*2

STANDARD*N



“Minimal Chaos”